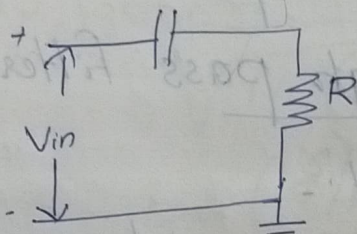


RC Circuits.

Combination of resistors and capacitors in a circuit forms RC circuits. Waveshaping may be defined as a process of generating new waveforms from older waveforms by employing certain physical systems. Linear waveshaping involves passage of signal through linear systems such as RC, RL, RLC circuits and the operations involved are linear such as integration, differentiation, summation, filtering etc.

High Pass RC Circuit.

The reactance of the capacitor is given by

$$X_C = \frac{1}{2\pi fC} \quad \text{or} \quad X_C \propto \frac{1}{f}$$

i.e. Reactance X_C of the circuit capacitor decreases with the increase in frequency. Thus at low frequencies the capacitor offers considerable reactance and so blocks them, but at higher frequencies the capacitor offers little reactance and allows them to pass through it. The output voltage is developed across resistor R .

With the increase in frequency, the reactance of the capacitor decreases and therefore the output and gain increase. At very high frequencies, the capacitive reactance becomes very small, so

Output becomes almost equal to the input and gain become equal to unity. Since this circuit attenuates the low frequency signals and allows transmission of high frequency signals with little or no attenuation.

Because of its blocking property for dc or low frequency input signals the capacitor acts like open circuit and blocks the signal. This is the reason that the capacitor is called the blocking capacitor. This circuit is widely employed as a coupling circuit to provide isolation between input and output.

Response of high pass filter:

Sine wave Input:-

For a sinusoidal input the waveshape of the output will remain the same; the output will differ only in amplitude and phase angle depending on the frequency of input signal.

Gain $A = \frac{V_{out}}{V_{in}}$

Current through circuit

$$\underline{I} = \frac{V_{in}}{R - jX_c}$$

Output voltage

V_{out} = Voltage drop across R

$$V_{out} = \underline{I} \cdot R = \frac{R \cdot V_{in}}{R - jX_c}$$

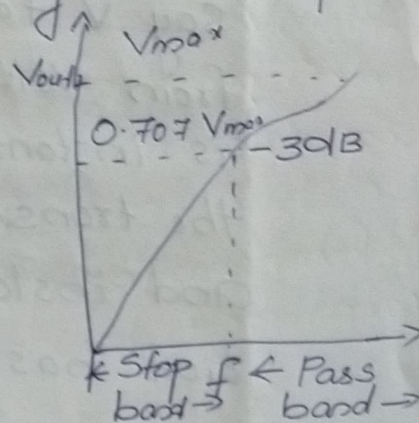
$$A = \frac{V_{out}}{V_{in}} = \frac{R}{R - jX_C} = \frac{1}{1 - j \frac{X_C}{R}}$$

A depends upon the frequency of the input signal

$$f_1 = \frac{1}{2\pi RC}$$

Substituting $f_1 = \frac{1}{2\pi RC}$

$$\text{Gain } A = \frac{1}{1 - j(f_1/f)}$$



Magnitude of A

$$|A| = \frac{1}{\sqrt{1 + (f_1/f)^2}}$$

$$\theta = \tan^{-1}(f_1/f)$$

As $f \rightarrow \infty$, $f_1/f = 0$ and $|A| \rightarrow 1$

$$\text{When } f = f_1, |A| = \frac{1}{\sqrt{2}} = 0.707$$

f_1 is also known as low 3-dB frequency.

For a 10% change in capacitor voltage, the time or pulse width is

$$t = 0.1RC = \text{pulse width}$$

$$\frac{PW}{PC} = 0.1$$

$$= 2\pi f_1 PW$$

$$V_{out} = V_{final} - (V_{final} - V_{initial}) e^{-t/\tau}$$

Step Input:-

A step voltage is zero for all times when $t < 0$ and maintains a voltage level say V for all times when $t \geq 0$. The transition from voltage level 0 to V occurs at $t = 0$ and is instantaneous. Instant of time just before the transition occurs is denoted by $t = 0^-$ and instant of time just after the transition has taken place is denoted by $t = 0^+$.

At instant $t = 0^-$ since the input is zero the output is zero. At instant $t = 0$, the input suddenly becomes V . Since the voltage across a capacitor cannot change instantaneously, the output voltage remains V . At $t = 0^+$ the input voltage remains unchanged at V , the capacitor starts charging exponentially with time constant R_c .

$$\text{Input voltage } V_{in} = 0 \text{ for } t < 0. \\ = V \text{ for } t \geq 0.$$

$$\text{Output voltage } V_{out} = iR.$$

Since at any instant input voltage

$$V_{in} = iR + \frac{1}{C} \int i \cdot dt$$

Taking Laplace

$$V_{in}(s) = R(s) + \frac{i(s)}{C(s)}.$$

Assuming zero charge on capacitor at $t = 0$

$$i(s) = \frac{V_{in}(s)}{R + \frac{1}{Cs}}$$

$$\text{and } V_{out}(s) = R \cdot i(s) = \frac{V_{in}(s)}{R + \frac{1}{Cs}} \times R$$

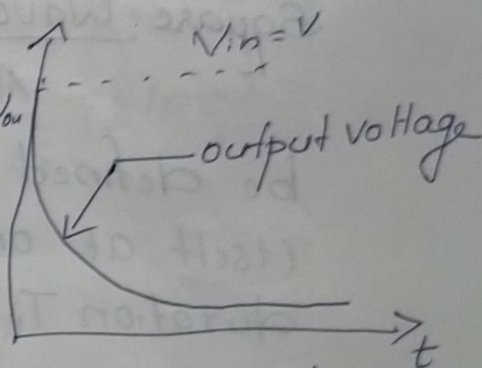
$$= \frac{V_{in}(s)}{1 + \frac{1}{RCs}} = \frac{V_{in}}{s + \frac{1}{RC}} \quad \because V_{in}(s) = \frac{V}{s}$$

Taking inverse Laplace we have

$$V_{out}(t) = V \cdot e^{-t/RC} = V \cdot e^{-t/\tau}$$

τ = time constant of circuit = RC .

The output voltage is equal to input voltage or initial value at $t=0$; 0.607 times of initial value at $t=0.5\tau$.



The voltage across a capacitor can change instantaneously only when an infinite current pass through it.

Pulse Input:-

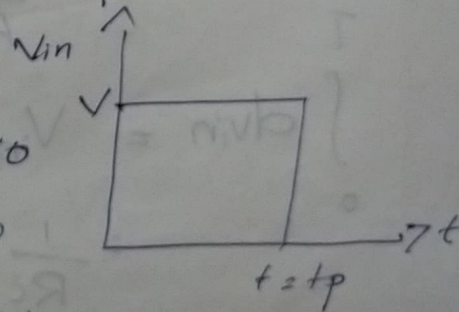
An ideal pulse voltage with amplitude V and pulse duration t_p

Input Voltage

$$V_{in} = 0 \text{ for } t < 0$$

$$= V \text{ for } 0 \leq t < t_p$$

$$= 0 \text{ for } t > t_p$$



For $t > 0$; input is zero and the output is also zero.

$$V_{out} = V \cdot e^{-t/Rc}$$

At $t = t_p$

$$V_{out} = V \cdot e^{-t_p/Rc}$$

$$V_{out} = V \cdot e^{-t_p/Rc} - V = V \left[e^{-t_p/Rc} - 1 \right]$$

The rate of exponential fall or rise depends upon the time constant Rc .

Square Wave Input:

A square wave may alternatively be defined as a waveform which maintains itself at one constant level V_1 for a time duration T_1 and another constant V_2 for time T_2 and $T = T_1 + T_2$.

Since at any instant

$$V_{in} = \frac{1}{C} \int i \, dt + V_{out}$$

$$\frac{dV_{in}}{dt} = \frac{i}{C} + \frac{dV_{out}}{dt}$$

$$= \frac{V_{out}}{Rc} + \frac{dV_{out}}{dt}$$

$$\int_0^T dV_{in} = V_{in}(T) - V_{in}(0)$$

$$= \frac{1}{Rc} \int_0^T V_{out} \cdot dt + V_{out}(T) - V_{out}(0)$$

Under Steady State Conditions

$$V_{out}(T) = V_{out}(0) \text{ and } V_{in}(T) = V_{in}(0)$$

$$\text{So } \int_0^T V_{out} \cdot dt = 0$$

Above integral represents the area under the output waveform over one cycle

Analysis:

For a constant voltage applied to the circuit, the output at any given instant is

$$V_{out} = V e^{-t/RC}$$

$$\text{At time } t = T_1, V_{out} = V_1 e^{-T_1/RC}$$

$$V_1' = V_1 e^{-T_2/RC}$$

$$\text{At } t = T_1 + T_2, V_2 = V_2 e^{-T_2/RC}$$

$$V_1 - V_2' = V$$

$$T_1 = T_2 = T/2, V_1 = -V_2$$

$$\text{and } V_1' = -V_2'$$

Substituting these values in above equation

$$V_1' = V_1 e^{-T_1/RC}$$

$$V_1' + V_1 = V$$

Eliminating V_1' we have

$$V_1 = V - V_1 e^{-T_1/RC}$$

$$V_1 [1 + e^{-T_1/RC}] = V$$

$$V_1 = \frac{V}{1 + e^{-T/2RC}}$$

For $\frac{T}{2RC} \ll 1$

$$V_1 = \frac{V}{1 + 1 - \frac{T}{2RC}}$$

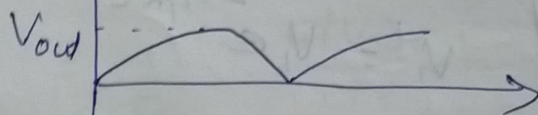
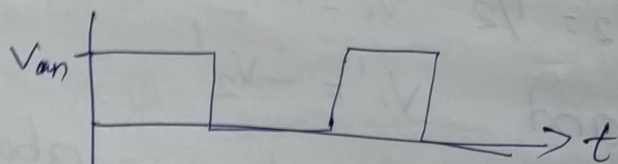
$$V_1 = \frac{V}{2 - \frac{T}{2RC}}$$

$$V_1 = \frac{V}{2} \left[1 - \frac{T}{4RC} \right]^{-1} = \frac{V}{2} \left[1 + \frac{T}{4RC} \right]$$

Similarly $V_1' = \frac{V}{2} \left[1 - \frac{T}{4RC} \right]$

$$P = \frac{V_1 - V_1'}{V/2} \times 100 = \frac{T}{2RC} \times 100$$

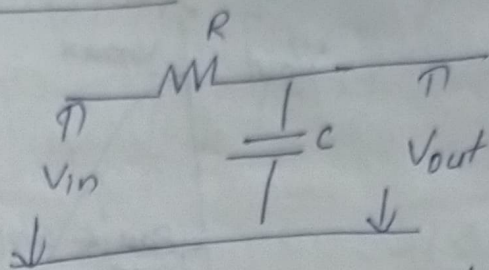
$$\therefore P = \pi \frac{f_1}{f} \times 100 \quad \therefore f_1 = \frac{1}{2\pi RC}$$



Applications:-

- RC coupling of amplifiers
- Triggering circuit.

Low Pass RC Circuit:-



In a low pass RC circuit, the output is taken across a capacitor. Resistor offers fixed opposition. Since reactance offered by capacitor C falls with the increase in frequency, low frequency signal develops across the capacitor but signal of frequency above cut off f_c develops negligible voltage across capacitor C .

Sinusoidal Input:-

For a sinusoidal input the waveshape of the output will remain the same; the output will differ only in amplitude and phase angle depending on frequency of the input signal.

$$\text{Gain } A = \frac{V_{out}}{V_{in}}$$

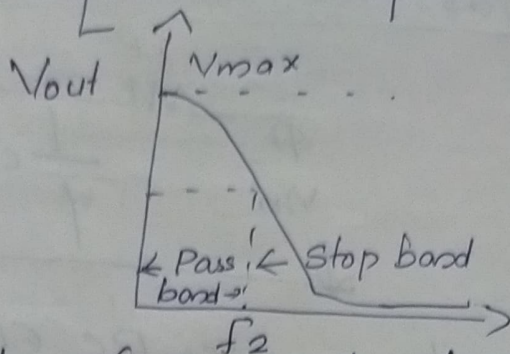
Current through the circuit is given as

$$I = \frac{V_{in}}{R - jX_C}$$

$$\text{Output Voltage } V_{out} = -jI X_C = \frac{-jX_C}{R - jX_C} \cdot V_{in}$$

$$\begin{aligned} \text{So gain } A &= \frac{V_{out}}{V_{in}} = \frac{-jX_C}{R - jX_C} = \frac{1}{1 - \frac{R}{jX_C}} \\ &= \frac{1}{1 + j2\pi fCR} \end{aligned}$$

From above equation response will be.



Now magnitude of A and phase angle θ

$$|A| = \frac{1}{\sqrt{1 + (f/f_2)^2}}$$

$$\theta = \tan^{-1}(f/f_2)$$

When $f = f_2$; $|A| = \frac{1}{\sqrt{2}} = \underline{\underline{0.707}}$

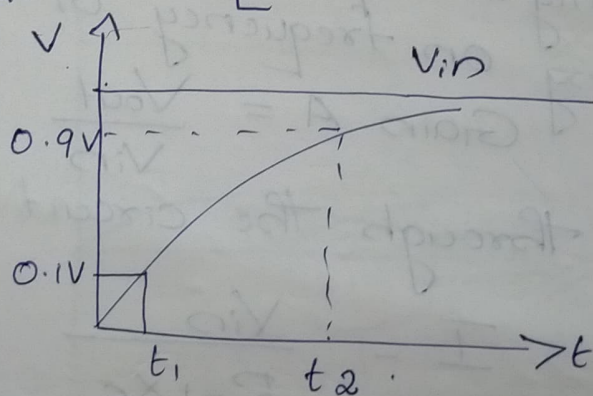
Step Input:-

At instant $t = 0^-$ Since input is zero, the output is also zero.

At instant $t = 0$; the input suddenly becomes V_{in}

At $t = 0^+$, the input voltage remains unchanged

$$V_{out} = V \left[1 - e^{-t/CR} \right]$$



Response to step.

Let the time required to attain $1/10^{th}$ of final value

$$V_{out} = 0.1V$$

$$0.1V = V[1 - e^{-t_1/RC}]$$

$$t_1 = -RC \log_e(0.9) \approx \underline{\underline{0.1RC}}$$

At time t_2

$$0.9V = V[1 - e^{-t_2/RC}]$$

$$e^{-t_2/RC} = 0.1$$

$$t_2 = -RC \log_e(0.1) = \underline{\underline{2.3RC}}$$

Rise time $t_r = t_2 - t_1$

$$= 2.3RC - 0.1RC$$

$$= 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2}$$

$$\Delta = f_2 - f_1$$

Bandwidth $\Delta f = f_2$

Thus rise time is proportional to time Constant 2.

Pulse Input :

The response to a pulse, for time less than pulse width t_p is same as Step input

The output for pulse input

$$V_{out} = V(1 - e^{-t/RC}) \text{ for } t < t_p$$

$$V_{out} = V(1 - e^{-t_p/RC}) = V_p$$

At $t = t_p$

$$V_{out} = V_p e^{-(t-t_p)/RC}$$

A pulse shape will be maintained if 3-dB frequency is approximately equal to reciprocal of pulse width.

Square Wave Input:

Consider a periodic waveform whose instantaneous value is constant at V' for a time T_1 and then changes abruptly. The input appears pulses of opposite polarity and added alternately.

$$V_{out} = V'(1 - e^{-t/RC})$$

$$\text{At } t = T_1 \quad V'(1 - e^{-T_1/RC})$$

Let at $t = 0^+$ capacitor is initially charged to a voltage V

$$\text{Input voltage } V_{in} = V'$$

Applying Kirchhoff's Voltage law

$$\frac{V'}{s} - Ri(s) - \frac{i(s)}{Cs} - \frac{V_1}{s} = 0.$$

$$i(s) = \frac{V' - V_1}{s \left[R + \frac{1}{Cs} \right]}$$

$$V_{out}(s) = \frac{i(s)}{Cs} + \frac{V_1}{s}$$

$$= \frac{V' - V_1}{Cs^2 \left[R + \frac{1}{Cs} \right]} + \frac{V_1}{s}$$

$$= \frac{V'}{s} + \frac{(V_1 - V')RC}{RCs + 1}$$

Taking Inverse Laplace

$$V_{out}(t) = V' + (V_1 - V')e^{-t/RC}$$

$$V_{out1}(t) = V' + (V_1 - V')e^{-t/RC}$$

$$V_{out2} = V'' + [(V_2 - V'')e^{-(t-T_1)/RC}]$$

Since at $t = T_1$ $V_{out1} = V_2 = V' + (V_1 - V')e^{-T_1/RC}$

For Symmetrical wave

$$V_1 = -V_2 ; V' = -V'' \text{ and } T_1 = T_2 = T/2$$

$$V_1 = -V' + (-V_1 + V')e^{-T/2RC}$$

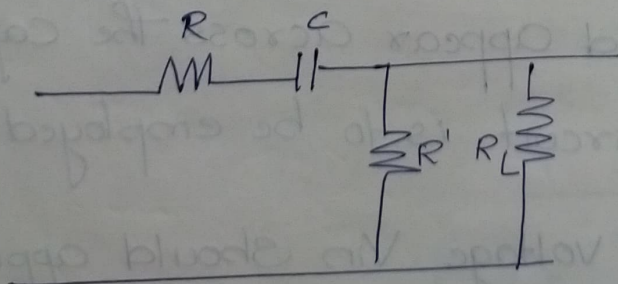
$$V_1 = \frac{V' \left[e^{-T/2RC} - 1 \right]}{e^{-T/2RC} + 1} = \frac{V}{2} \frac{\left[e^{-T/2RC} - 1 \right]}{\left(e^{-T/2RC} + 1 \right)}$$

$$V_2 = -\frac{V}{2} \frac{\left[e^{-T/2RC} - 1 \right]}{\left[e^{-T/2RC} + 1 \right]}$$

Rc. Differentiator:-

A circuit that gives an output voltage proportional to the derivative of its input is known as differentiating circuit

$$\text{Output} \propto \frac{d}{dt} [\text{Input}]$$



The time constant RC of the circuit should be much smaller than the time period of the input signal $RC \ll T$

The value of X_C should not be smaller than 10 times of R i.e. $X_C \geq 10R$.

If V_{in} is alternating voltage, i is the resulting alternating current then instantaneous charge on the capacitor is

$$q = C V_C$$

$$\text{Current } i = \frac{dq}{dt} = \frac{d}{dt} (C \cdot V_C) = C \cdot \frac{dV_C}{dt}$$

As the capacitive reactance is much larger than $X_C \geq 10R$

$$i = C \cdot \frac{d}{dt} V_{in}$$

$$\text{Output Voltage } V_{out} = iR = RC \cdot \frac{d}{dt} V_{in}$$

$$V_{out} \propto \frac{d}{dt} V_{in}$$

If the input voltage is the square wave, output wave should be impulses of infinite amplitude and alternating polarity. Thus if the R-C circuit is to be employed as a differentiator, input voltage V_{in} should appear across the capacitor.

Thus if the R-C circuit is to be employed as a differentiator, input voltage V_{in} should appear across C .

Module - II

Small Signal Analysis of CE, CB, CC Configuration using small hybrid π -model.

The overall transistor circuit will work as linear system if the transistor of the circuit behaves as a linear device. If the transistor circuits are operated by the class of small signal low frequency input signals, the transistor can be modelled as a two port network. The ac base emitter voltage is $V_{be} = r_e' i_e$. The model yields $i_c = g_m V_{be}$ and $i_b = V_{be} / r_{\pi}$.

$$i_e = \frac{V_{be}}{r_{\pi}} + g_m V_{be} = \frac{V_{be}}{r_{\pi}} [1 + g_m r_{\pi}]$$

$$= \frac{V_{be}}{r_{\pi}} (1 + \beta) = V_{be} / \frac{r_{\pi}}{(1 + \beta)}$$

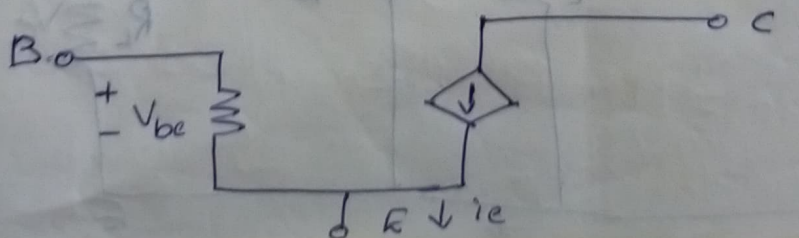
$$= \frac{V_{be}}{r_e}$$

A slightly different equivalent circuit model

$$g_m V_{be} = g_m (i_b r_{\pi})$$

$$= (g_m r_{\pi}) i_b = \underline{\underline{\beta i_b}}$$

This results in the alternative equivalent circuit. Here the transistor



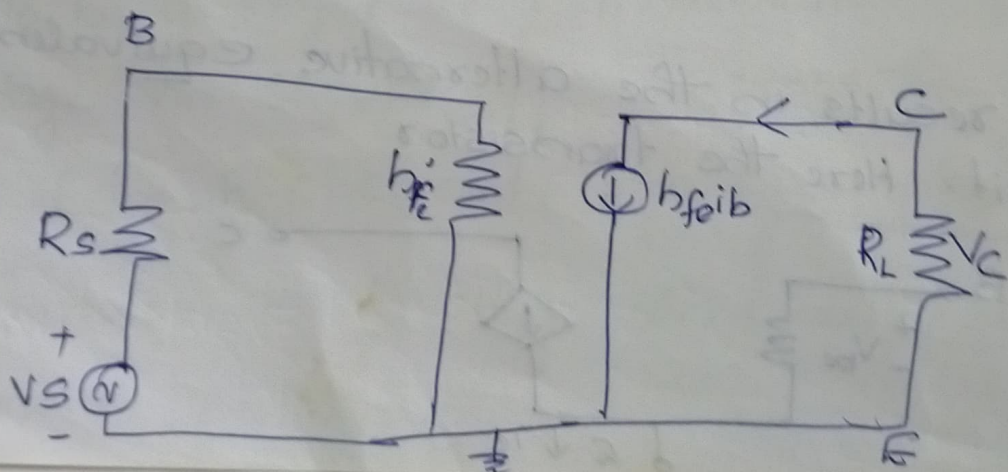
Application of Small Signal Equivalent Circuit

The availability of the small signal BJT circuit models makes the analysis of transistor amplifier circuits a systematic process.

1. Determine the dc operating point of the BJT and in particular the dc collector current I_C .
2. Calculate the values of small signal model parameters $g_m = I_C/V_T$, $r_{\pi} = \beta/g_m$ and $r_e = V_T/I_E = \alpha/g_m$.
3. Eliminate the dc sources by replacing each dc voltage source with a short circuit and each dc current source with an open circuit.
4. Replace BJT with small signal equivalent circuit models.
5. Analyse the resulting circuit to determine the required quantities.

Analysis of Common Emitter [CE - Amplifier]

CE is most commonly used configuration in BJT amplifier circuits. To establish a signal ground, at emitter



On the output side the resistance $\frac{1}{h_{oe}}$ appears in parallel with the load resistance R_L . Under this condition, the magnitude of voltage of generator in emitter circuit

$$h_{fe} V_c = h_{re} I_c R_L = h_{re} h_{fe} I_b R_L$$

Current Gain :-

The signal is connected between the input and ground terminals and load is connected between output and ground terminals

$$A_i = - \frac{h_{fe}}{1 + h_{oe} Z_L}$$

Input Impedance

$$Z_{in} = h_{ie} + h_{re} \frac{V_2}{I_1}$$

$$= h_{ie} + h_{re} \frac{A_i I_1 Z_L}{I_1}$$

$$= h_{ie} + h_{re} A_i Z_L$$

$$Z_{in} = h_{ie} \left[1 - \frac{h_{re} h_{fe} / A_i / h_{oe} Z_L}{h_{ie} h_{oe} h_{fe}} \right]$$

$$|A_i| = \frac{h_{fe}}{1 + h_{oe} R_L} \approx h_{fe}$$

$$Z_{in} = h_{ie} \left(1 - \frac{0.5 h_{fe} h_{oe} Z_L}{h_{fe}} \right)$$

$$= h_{ie} \approx \frac{V_b}{I_b}$$

Voltage Gain:-

Ratio of output voltage V_2 and input voltage V_1 gives the voltage gain

$$A_V = \frac{V_2}{V_1} = -\frac{I_2 Z_L}{V_1} = \frac{A_i I_1 Z_L}{V_1}$$

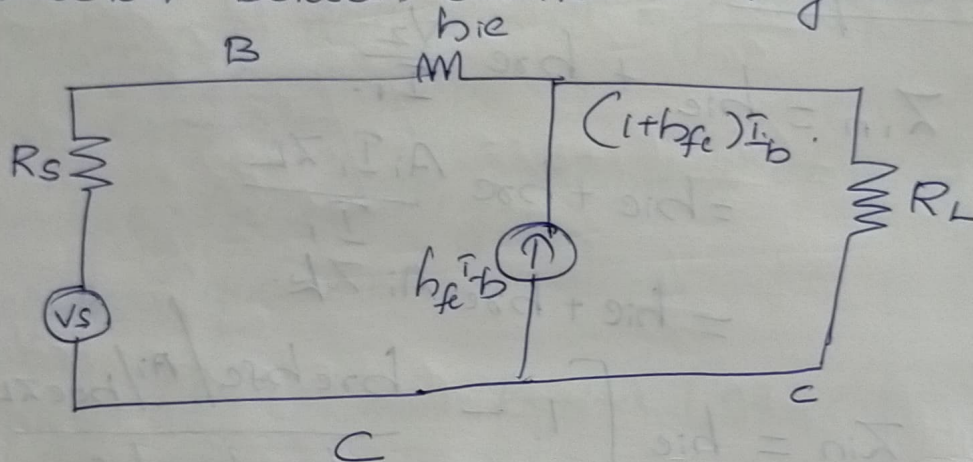
$$= \frac{A_i Z_L}{Z_{in}} = -\frac{h_{fe} Z_L}{h_{ie}}$$

Output Impedance:-

Output impedance will be zero.

Common Collector Configurations:-

Collector is grounded and load R_L is connected between emitter and ground.



$$A_I = -\frac{I_E}{I_b} = 1+h_{fe}$$

Input Resistance

$$R_i = \frac{V_b}{I_b} = h_{ie} + (1+h_{fe}) R_L$$

$R_i \gg h_{ie} \approx 1\text{K}$ R_L is small as 0.5K
because $h_{fe} \gg 1$.

Output Impedance -

$$I = (1 + h_{fe}) \bar{I}_b = \frac{(1 + h_{fe}) V_s}{h_{ie} + R_s}$$

Output admittance of transistor

$$I = (1 + h_{fe}) \bar{I}_b = \frac{(1 + h_{fe}) V_s}{h_{ie} + R_s}$$

Hence output admittance of the transistor

$$Y_o = \frac{I}{V_s} = \frac{1 + h_{fe}}{h_{ie} + R_s}$$

$$Y_o = h_{oe} + \frac{1 + h_{fe}}{h_{ie} + R_s}$$

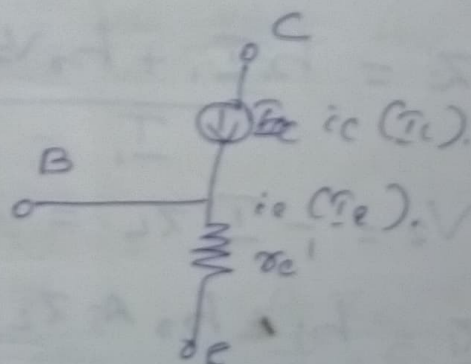
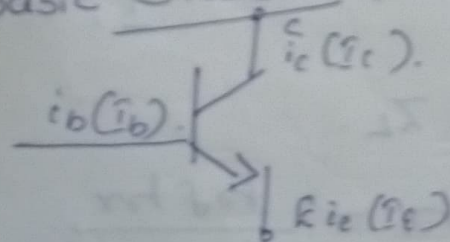
$$R_o = \frac{h_{ie} + R_s}{1 + h_{fe}}$$

Small Signal Analysis of BJT amplifier Circuits

There are three basic configurations for implementing BJT amplifiers.

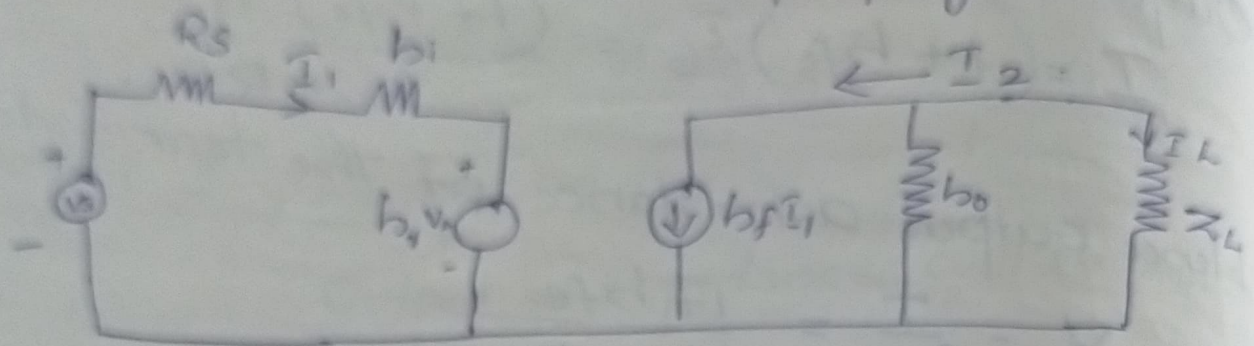
- Common Emitter
- Common Base
- Common Collector

Basic Structure



Analysis of a Transistor Amplifier Circuit using h-parameters

To form a transistor amplifier, it is only necessary to connect an external load and signal source to bias the transistor properly.



A_I is defined as the ratio of output to input currents

$$A_I = \frac{I_L}{I_1} = -\frac{I_2}{I_1}$$

$$I_2 = h_{fe} I_1 + h_{oe} V_2$$

$$A_I = -\frac{I_2}{I_1} = -\frac{h_{fe}}{1 + h_{oe} Z_L}$$

Input Impedance:-

$$\text{Input impedance } Z_i = \frac{V_1}{I_1}$$

$$V_1 = h_{ie} I_1 + h_{re} V_2$$

$$Z_i = \frac{h_{ie} I_1 + h_{re} V_2}{I_1} = h_{ie} + h_{re} \frac{V_2}{I_1}$$

$$V_2 = -I_2 Z_L = A_I I_1 Z_L$$

$$Z_i = h_{ie} + h_{re} A_I Z_L = h_{ie} - \frac{h_{fe} h_{re}}{h_{fe} + h_{oe}}$$

Load admittance $Y_L = 1/Z_L$

Voltage Gain:

Ratio of output voltage V_2 to input voltage V_1 gives the voltage gain of transistor.

$$A_V = \frac{V_2}{V_1}$$

$$A_V = \frac{A_I I_1 Z_L}{V_1} = \frac{A_I Z_L}{Z_i}$$

Output Admittance:

$$Y_o = \frac{I_2}{V_2} \Big|_{V_s=0}$$

$$Y_o = h_{fe} \frac{I_1}{V_2} + h_{oe}$$

$$R_s I_1 + h_{ie} I_1 + h_{re} V_2 = 0$$

$$\frac{I_1}{V_2} = -\frac{h_{re}}{h_{ie} + R_s}$$

$$Y_o = h_{oe} - \frac{h_{fe} h_{re}}{h_{ie} + R_s}$$

Output admittance is a function of source resistance.

Overall Voltage gain

$$A_{Vs} = \frac{V_2}{V_s} = \frac{V_2}{V_1} \frac{V_1}{V_s} = A_V \frac{V_1}{V_s}$$

$$V_1 = \frac{V_s Z_i}{Z_i + R_s}$$

$$A_{Vs} = \frac{A_V Z_i}{Z_i + R_s} = \frac{A_I Z_L}{Z_i + R_s}$$

Current Amplification:

$$A_{I_s} = \frac{-I_2}{I_s} = -\frac{I_2}{I_1} \cdot \frac{I_1}{I_s} = A_I \frac{I_1}{I_s}$$

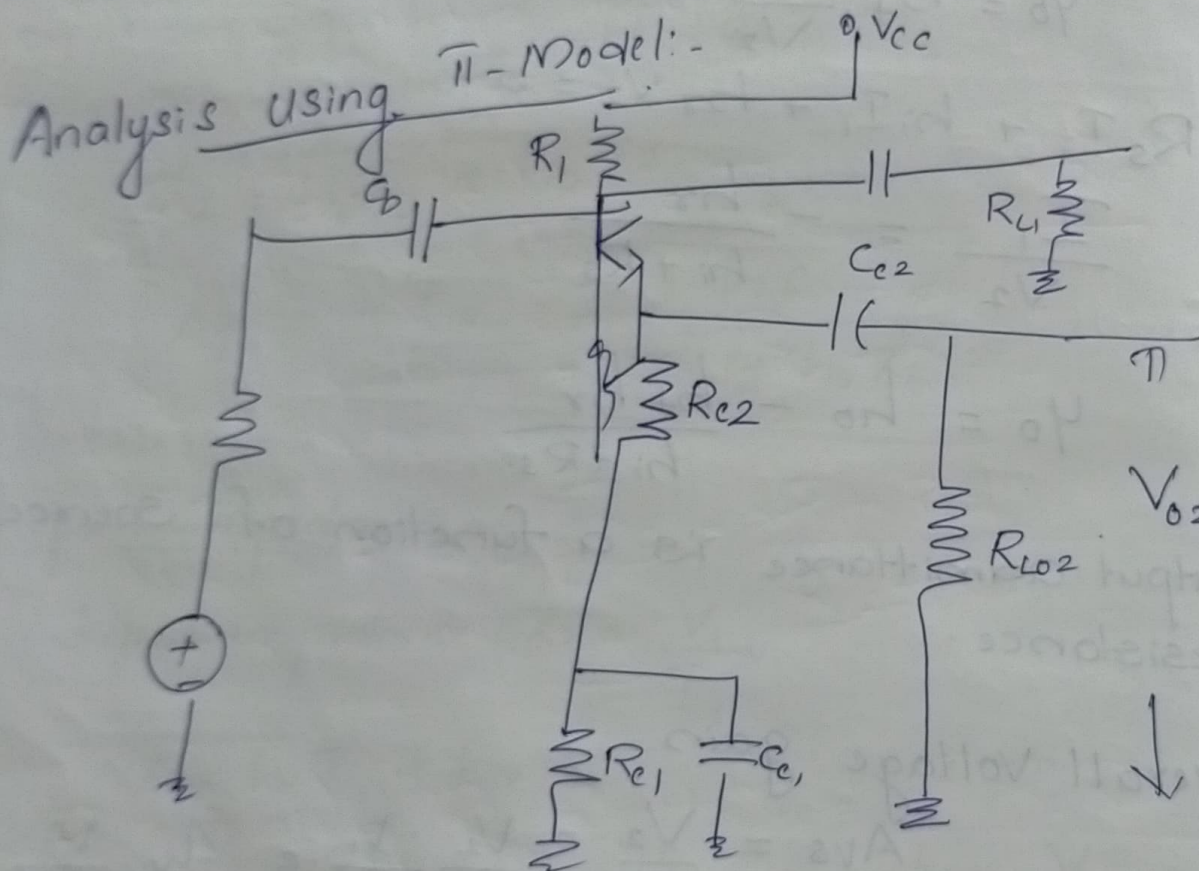
$$I_1 = \frac{I_s \cdot R_s}{Z_i + R_s}$$

$$A_{I_s} = \frac{A_I R_s}{Z_i + R_s}$$

A_I is current gain.

Power Gain:

$$A_p = \frac{P_2}{P_1} = -\frac{V_2 I_2}{V_1 I_1} = A_I^2 \frac{R_L}{R_i}$$



Thevenin voltage at input side

$$V_{th} = \frac{R_1 \parallel R_2}{R_s + (R_1 \parallel R_2)}$$

$$R_{th} = R_s \parallel R_1 \parallel R_2$$

Let R_{L1c} and R_{L2e} be the resistance s of parallel combination R_c and R_{L1} and R_{e2} and R_{L2}

Thus we write

$$R_{L1c} = R_c \parallel R_{L1} = \frac{R_c R_{L1}}{R_c + R_{L1}}$$

$$R_{L2e} = R_{e2} \parallel R_{L2} = \frac{R_{e2} R_{L2}}{R_{e2} + R_{L2}}$$

Applying KVL in input side

$$V_{th} - I_b R_{th} + I_b h_{fe} r_{e'} + I_b h_{fe} R_{L2e} = 0$$

which gives the base current as

$$I_b = \frac{V_{th}}{R_{th} + h_{fe} (r_{e'} + R_{L2e})}$$

$$= V_s \left[\frac{1}{R_{th} + h_{fe} (r_{e'} + R_{L2e})} \cdot \frac{R_c R_{L1}}{R_c + R_{L1}} \cdot \frac{(R_1 \parallel R_2)}{R_s + (R_1 \parallel R_2)} \right]$$

$$V_{o2} = R_{L2e} I_c \approx R_{L2e} I_c$$

$$= \left[\frac{h_{fe} R_{e2} R_{L2} (R_1 \parallel R_2)}{R_{th} + h_{fe} (r_{e'} + R_{L2e}) R_s + (R_1 \parallel R_2)} \right] V_s$$

$$A_{Vs I_s} = \frac{V_{o1}}{I_s} = \frac{-h_{fe} R_c R_{L1} (R_1 \parallel R_2)}{R_{ib} + h_{fe} (\tau_{e1}' + R_{12e}) (R_c + R_{L1}) + (R_1 \parallel R_2)}$$

$$A_{V1} = \frac{V_{o1}}{V_{in}} = \frac{-R_{e1c} h_{fe} I_b}{I_b h_{fe} (\tau_{e1}' + R_{12e})} = \frac{-R_{e1c}}{(\tau_{e1}' + R_{12e})}$$

Input Resistance

The input resistance or the input impedance of a network is the resistance between input terminals seen by a source, when it is connected to the circuit.

Let R_{in} be the input resistance at the base seen by input signal V_{in} appearing between the base and ground terminals.

$$R_{in}(\text{base}) = \frac{V_{in}}{I_b} = \frac{I_b h_{fe} (\tau_{e1}' + R_{12e})}{I_b}$$

$$= h_{fe} [\tau_{e1}' + R_{12e}]$$

$$= h_{fe} \left[\tau_{e1}' + \frac{R_{e2} R_{L2}}{R_{e2} + R_{L2}} \right]$$

$$R_{in}(\text{total}) = R_1 \parallel R_2 \parallel R_{in} = \frac{R_1 R_2 R_{in}}{R_2 R_{in} + R_1 R_{in} + R_1 R_2}$$

Current Gain:-

is defined as ratio of output current to the input current.

$$\underline{I}_s = \frac{V_s}{R_s + R_{in}(\text{total})}$$

$$A_{Isc} = \frac{I_c}{I_s} \approx \frac{I_e}{I_s} = \frac{h_{fe} I_b}{I_s}$$

$$A_{Isc} = \frac{h_{fe} [R_s + R_{in}(\text{total})] (R_1 \parallel R_2)}{R_{th} + h_{fe} (r_{e'} + R_{12e}) R_s + (R_1 \parallel R_2)}$$

Output Resistance:-

The output resistance

$$R_{out} = \frac{\text{Output voltage with load open}}{\text{Output current with shorted load}}$$

$$R_{out1} = \frac{V_{o1} \text{ with } R_L = \infty}{I_c \text{ with } R_L = 0} = \frac{1}{h_{fe}} \frac{V_{o1} |_{R_L = \infty}}{I_b |_{R_L = 0}}$$

$$= \frac{1}{h_{fe}} \left[\frac{h_{fe} R_c (R_1 \parallel R_2) V_s}{R_{th} + h_{fe} (r_{e'} + R_{12e}) R_s + (R_1 \parallel R_2)} \right] \div \left[\frac{1}{R_{th} + h_{fe} (r_{e'} + R_{12e}) R_s + (R_1 \parallel R_2)} \right]^{-1} V_s$$

$$\underline{R_{out} = R_c}$$

$$R_{out2} = \frac{V_{o2} |_{R_{L2} = \infty}}{I_{e2} |_{R_{L2} = 0}}$$